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INTERPRETING FORMAL LOGIC*

ABSTRACT. The concept of *semantic interpretation* is a source of chronic confusion: the introduction of a notion of *interpretation* can be the result of several quite different kinds of considerations. *Interpretation* can be understood in at least three ways: as a process of "dis-abstraction" of formulas, as a technical tool for the sake of characterizing truth, or as a reconstruction of meaning-assignment. However essentially different these motifs are and however properly they must be kept apart, they can all be brought to one and the same notion of interpretation: to the notion of a compositional evaluation of expressions inducing a "possible" distribution of truth values among statements.

1 WHAT IS A *SEMANTIC INTERPRETATION*?

The concept of semantic interpretation might seem quite unproblematic. Expressions of our natural language stand for a kind of objects; therefore, if we make a logical formalization of language, we should make the expressions of the formal language also stand for something. Semantic interpretation is then what establishes the link between the formulas and what they stand for. According to this view, semantic interpretation is a formal imitation of the real denotandum/denotatum relation.

However, such a view, although accepted by many theoreticians of language, is utterly naive; it rests on the identification of language with a kind of nomenclature. We assume that expressions are sort of labels which we attach to pre-existing, real-world objects to make the objects capable of being referred to. Accordingly, our world is a great museum, the exhibits of which are waiting to be classified and named by us; therefore Quine (1969) calls this view the *museum myth*.

However, there are also other meanings in which we use the term *interpretation*. Even if we disregard the sense which underlies the enterprise of hermeneutics, there remain at least two other meanings, both of which are essential for formal logic and for the logical analysis of language. In one of these meanings interpretation is an assignment of concrete instances to items of an abstract formal system, typically an assignment of concrete language expressions to abstract formulas. Then there is the technical sense of *interpretation* used in textbooks of mathematical logic, where interpretation is regarded as a technical means of characterizing truth.[1]

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However, are these three meanings really essentially different? Some authors appear to feel free to pass from one to another (Tarski's, 1936, introduction of the concept of model is an example of a fluent passage from the second meaning to the third; while Cresswell's, 1973, easy-going switch from semantics to "metaphysics" does not seem to make any distinction between the third and the first meaning, nor do many other contemporary "metaphysical" considerations based on model theory). In order to be able to understand the proper role of the concept of interpretation within the enterprise of formal logic and logical semantics, we must first clarify the basic tenets of logical formalization.

2 TWO MODES OF DOING FORMALIZATION

Formal logic is based on the utilization of symbols. Symbols function as substitutes for natural language expressions; it is the utilizations of symbols that help us to ignore the irrelevant aspects of natural language expressions and to point out patterns relevant for consequence. Thus, the symbolization is that which helps us, as Frege (1879, p. IV) put it, "die Bündigkeit einer Schlu_kette auf die sicherste Weise zu prüfen und jede Voraussetzung, die sich unbemerkt einschleichen will, anzuzeigen, damit letztere auf ihren Ursprung untersucht werden könne."

However, symbols may be utilized in different ways. It is especially important to distinguish between two quite disparate modes of their employment, between the *regimentative mode* and the *abstractive mode*.

Employing symbols in the regimentative mode means no more than to disregard irrelevant peculiarities of grammar of natural language. To express the fact that John loves Mary we may use various natural language statements, e.g. *John loves Mary*, *It is John who loves Mary*, *Mary is loved by John*; however, on the level of logical schematization all these ways may boil down to canonical *loves(John,Mary)*, or, if we employ P to represent *loves*, T_1 to represent *John* and T_2 to represent *Mary*, to $P(T_1,T_2)$[2]. Hence regimentation means only the reduction of redundancies in the lexicon and/or grammar of natural language.

This means that regimentation is simply a kind of sifting of natural language through the sieve of relevance; what is relevant is unambiguously retained in the resulting formal representation, that which is not, vanishes. Symbols and their concatenations utilized for the purposes of regimentation may be truly called *constant*: each of these stands constantly for a definite natural language expression or at least for a definite "pattern" common to several synonymous natural language expressions.

The other mode of employment of symbols - abstraction - is of a different nature. While doing regimentation we do not abandon the level of concrete expressions, abstraction leads to articulating *types* of expressions. We may use the symbol P to represent an *arbitrary* binary predicate and the symbols T_1 and T_2 to represent *arbitrary* terms; $P(T_1,T_2)$ then represents *every* statement which shares the form with *John loves Mary*.

Symbols employed in the abstractive mode may be looked at in varying ways; T_1 used as a means of abstraction can be considered once as *John*, once as *Mary*, etc. The symbols are thus not constants in the proper sense of the word, they are rather a kind of *parameters*. In contrast to constant formal expressions, formal expressions containing such parameters shall be called *parametric* formal expressions[3]; those parametric formal expressions which represent statements also will be called *schemata*. Hence a schema does not stand for a concrete natural language statement, it is a mere matrix.

If we now look at standard logic, we can distinguish two kinds of symbols. There are symbols that are used unequivocally in the regimentative mode. The examples of these symbols are logical connectives, quantifiers, or the equality sign. Such a symbol as **&** is surely not meant to be considered once as *and* and once as *or*, it is meant to represent a definite way of conjoining statements, a way which is, in the prototypical case, expressed by *and*. Symbols of this kind are usually called *logical constants*.

The other symbols, called *extralogical constants*, are ambiguous between constants proper and parameters. Their examples are nonspecific terms or predicates. The term constant T_1, for example, can be understood as being a constant proper (representing, e.g., the name *John*), or it can be understood to be a mere schematic representation of an arbitrary term. This ambiguity then extends to all statements containing extralogical constants: $P(T_1,T_2)$ may be understood to represent a concrete statement, such as *John loves Mary*, or it can be understood as an abstract schema amounting to all statements of the relevant form. The reason why we can treat extralogical, in contrast to logical, constants in this way is that properties of statements which are interesting from the point of view of logic (especially their behaviour from the viewpoint of consequence) are invariant under the replacement of an extralogical constant by another one. Thus, we can replace T_1 by any other term in an instance of consequence without disturbing its "consequencehood" (while we *cannot* so replace, e.g., **&** by another logical connective).

The fact that it is not always fully clear in which sense some of the symbols of the languages of formal logic are employed is the reason for a profound ambiguity. We may view the predicate calculus (and indeed any other formal calculus) in two ways: the first view, the regimentative view, is that every statement of the calculus is constant, that it stands for a definite natural statement (although when it contains extralogical constants, we need not and do not say for which natural statement it stands); the second view, the abstractive view, is that each statement containing extralogical constants is a mere schema, that it amounts to all those natural language statements that conform to it.

3 INTERPRETATION AS "DIS-ABSTRACTION"

The first notion of interpretation we are going to address is based on the abstractive view. If we understand a formula as an abstract schema, then the formula, and any argument or proof including it, covers a multiplicity of individual instances. If we say that, e.g., $P(T_1,T_2)$ entails $\exists x.P(x,T_2)$, then what we say is that *John loves Mary* entails *Someone loves Mary*, that *Peter hates Jane* entails *Someone hates Jane*, etc. If reasoning about John's loving Mary, I can use the abstract schema understanding P as *loves*, T_1 as *John* and T_2 as *Mary*, if what I have in mind is Peter's hating Jane, then I may use it understanding P as *hates*, T_1 as *Peter* and T_2 as *Jane*. In other words, I may *interpret* P as *loves*, T_1 as *John* and T_2 as *Mary*, or P as *hates*, T_1 as *Peter* and T_2 as *Jane*. This leads to the first sense of *interpretation*, which can be called *interpretation as "dis-abstraction"*; dis-abstraction consists in pinning down parameters to constants, it is the assignment of constants to parameters (or constants proper to extralogical constants).

Any assignment of constants to parameters induces an assignment of a constant expression to every parametric one, especially an assignment of a constant statement to every schema. Every schema is assigned one of its instances. Such an interpretation represents one of many possible "temporary" systematic identifications of abstract entities (parametric expressions) with their concrete instances (constant expressions).

This notion of dis-abstraction is thus quite straightforward; however, only up to the point when we try to do justice to our intuitive view that the ultimate instances of abstract formulas are concrete *things* rather than concrete expressions; that instances of $P(T_1,T_2)$ are not the statements *loves(John, Mary)*, *hates(Peter, Jane)*, etc., but rather the *facts* of John's loving Mary, of Peter's hating Jane, etc. But to embrace this intuition seems to be the only way to gain the notion of dis-abstraction which is not trivially dependent on the resources of a particular language: there may clearly be an "obejctual" instance without there being a corresponding substitutional instance; we may imagine that Peter loves someone for whom we have no name.

From this point of view it seems to be appropriate to consider interpretation as not an assignment of constants, but rather of things (some of which are denotations of constants and others possibly not). However, this way we face the problem of identifying the definite real-world objects which expressions stand or may stand for; the problem that has been recognized as essentially tricky. How should we investigate the world in order to find out the "objectual" instances of $P(T_1,T_2)$? Is John's yesterday's loving Mary an instance other than his loving her today? Is an instance's instantiating the formula itself an instance? To answer these questions by thinking about the world means to do speculative metaphysics; and was it not just this kind of speculative metaphysics that had to be overcome by means of logical analysis of language?

4 INTERPRETATION AS CHARACTERIZATION OF TRUTH

An interpretation, as just revealed, maps every schema on a constant statement; and as every constant statement has a truth value, an interpretation induces a mapping of schemata on truth values. If a schema is mapped on truth in this manner, we shall say that it is *verified* by the interpretation; otherwise we shall say that it is *falsified*. Now if we switch from the abstractive mode to the regimentative mode, we cannot consider interpretation as a matter of schemata, but rather as a matter of constant statements; interpretation is then something that maps constants on constants, hence constant statements on other constant statements, and consequently constant statements on truth values.

Let us distinguish between true statements which are true *contingently* and those which are true *necessarily*. A statement is true contingently if it can be false; it is true necessarily if its falsity is impossible, if such a falsity is beyond the scope of our imagination[4]. Now, and this is of crucial importance, every necessarily true statement comes out verified by every interpretation. This follows from the way extralogical constants are chosen - they are the expressions which are freely interchangeable without disturbing consequence. Therefore, as consequence and necessary truth are merely two sides of the same coin[5], they are also freely interchangeable without disturbing necessary truth. This means that no permutation of extralogical constants can turn a necessary truth into a falsity, and therefore no interpretation can falsify a necessary truth. Hence verification by every interpretation is a necessary condition of necessary truth.

The idea behind the proposal of using interpretations for the purposes of characterization of necessary truth is to consider verification by every interpretation as not only a necessary, but also a sufficient condition of necessary truth. Such a proposal was clearly articulated by Tarski (1936); so let us call it *Tarski's thesis* [6]. If we call a statement verified by every interpretation *logically true*, then Tarski's thesis amounts to the identification of necessary and logical truth, or, better, to the explication of the concept of necessary truth (which is a natural, empirical concept) by means of the concept of logical truth (which is a technical, defined concept).

Let us stress that Tarski's proposal can be considered as a purely technical matter, as a general version of the *matrix method* as proposed (for the propositional calculus) e.g. by Lukasiewicz and Tarski (1930). The basis of this method is to consider certain collection of assignments of objects (prototypically of the two truth values *TRUE* and *FALSE*) to statements; some of the assigned objects are in a way distinguished (in the prototypical case it is the value *TRUE*) and a statement is declared logically true if it is always assigned a distinguished element. The matrix method, as Lukasiewicz and Tarski clearly saw, is simply an alternative to the axiomatic method of defining a formal calculus.

However, Tarski immediately realized that if we use the "substitutional" notion of interpretation considered so far, then there is no guarantee that logical truth would always come out identical with necessary truth; there is no reason to be sure that every statement verified by every substitutional interpretation will in fact be a statement intuitively understood as necessarily true. Let us consider *loves*, *John* and *Mary* as constants, and let us suppose that these three words are *the only* constants of our language; in that case every substitutional instance of $P(T_1,T_2)$ may be true (in the imaginable case when John loves Mary and himself and Mary loves John and herself), and hence *loves(John,Mary)* may be logically true, although John's love for Mary is surely not a necessary matter.

This means that if we accept Tarski's thesis, we are left with a kind of modification of the notion of interpretation: if there are some logical truths which are not necessary, then we have to postulate more interpretations and thus have to reduce the number of logical truths. The fact that *loves(John,Mary)* is not necessarily true implies (if we take Tarski's thesis for granted) that there must be an interpretation of $P(T_1,T_2)$ which falsifies it; if there is no such substitutional interpretation, we have to postulate interpretations beyond the substitutional ones.

If there is to be an interpretation of $P(T_1,T_2)$ beyond the four interpretations making it into *loves(John, Mary)*, *loves(Mary, John)*, *loves(John, John)*, *loves(Mary, Mary)*, then there must be either an instance of T_1 or T_2 other than *John* and *Mary*, or an instance of *P* other than *loves*. Anyway, there must be instances of parameters beyond expressions; there must be something other than constant expressions on which parameters can be mapped by interpretations.

The case of $P(T_1,T_2)$ can be solved by assuming that besides *John* and *Mary* there is a third instance for parametric terms, an instance that leads to the needed falsifying interpretation; for example that there is some X such that if T_2 is considered as this X then *loves(John,*T_2*)* comes out false. What is the nature of this X? It is not a constant term; for we have fixed our language to have *John* and *Mary* as the only two terms. We may think of it as of a *'potential' term* and assume that to consider it means to consider a potential extension of our language. However, the more straightforward way seems to be to give up the whole idea that term parameters are interpreted by constants, and to consider them as interpreted by some more abstract entities. The entities may be called *individuals*[7]. In our case we have to assume that we have three individuals: John, Mary and X.

The situation is similar in the general case. Whenever a statement is not necessarily true while it is verified by every interpretation, we need more interpretations, concretely we need an interpretation which would falsify the statement in question. But we need not make do with adding instances of parametric terms: e.g. in the above example the case is that every interpretation verifies $\exists x.\exists y.P(x,y)$ although such a statement as $\exists x.\exists y.loves(x,y)$ is not necessarily true, and this can be rectified only by postulating an instance of *P* other than *loves*. Concluding we cannot make do with constant predicates as instances, we must introduce new kinds of instances of parametric predicates and call them e.g. *relations* (*properties* in the unary case). In this way we reach a wholly new notion of interpretation that is no longer substitutional; it is *denotational*.

However, the passage from substitutional to denotational interpretation is not entirely unproblematic. The point is that every substitutional interpretation induces a mapping of schemata on truth values which makes it possible to talk about *verification* and *falsification* in connection with an interpretation and thus to make sense of Tarski's thesis; but if we pass from terms to individuals, from predicates to properties and relations, and from expressions to "abstract" objects in general, then the induction does not work any longer. To make it work we need the interpretations of parts of a whole to add up into an interpretation of the whole; furthermore we need the interpretation of statement somehow to yield a truth value (or directly *to be* a truth value). For an atomic statement (consisting of a predicate and terms), we need the relation interpreting the predicate together with the individuals interpreting the terms to add up into the interpretation of the statement and this interpretation either to have, or to be a truth value. The most straightforward way to achieve this is, of course, to identify n-ary relations with functions from n-tuples of individuals to truth values and to interpret an atomic statement by the value of the application of the interpretation of its predicate to the interpretations of its terms.

The concept of substitutional interpretation was based on the assignment of expressions to expressions; only extralogical constants had to be interpreted. Denotational interpretation is in this respect different: interpretation now means mapping of expressions on extralinguistic objects; hence all constants (logical as well as extralogical) have to be interpreted. Now the distinction between logical and extralogical constants may be considered to consist in the fact that a logical constant is always interpreted by one and the same object, whereas an extralogical constant may be interpreted by whatever object of the corresponding domain.

However, it appears that this kind of change is necessary independently of the passage from substitution to denotation. The point is that the sharp boundary drawn between logical and extralogical constants is in fact untenable. As Etchemendy (1988) points out, there are statements which consist purely of logical constants and which are, nevertheless, intuitively not necessary[8]. The picture saying that there are expressions whose interpretation is completely fixed, and that there are, on the other hand, expressions the interpretation of which is completely free (within the bounds of the corresponding domain), is oversimplified; there are in fact also expressions whose interpretation is partially fixed and partially free[9]. This fact can be naturally accounted for just by considering an interpretation an assignment of values to *all* expressions, some expressions (the purely logical ones) always being interpreted in the same way, others (the purely extralogical ones) quite freely, and the remaining ones in some partly limited way.

5 PROVISIONAL SUMMARY

There are assignments of truth values to statements which are "possible" and others which are "impossible". An assigment of truth values to statements is in this sense "impossible" iff it violates consequence (consequence can, in fact, be considered as delimitation of the space of "possible" truth valuations)[10]. Thus to say that something is necessarily true is to say that it is verified by every "possible" distribution of truth values. This implies that, as we need all and only necessary truths to come out as verified by every interpretation, that the class of distributions of truth values induced by interpretations should coincide with the class of "possible" distributions, that is, that an interpretation should be defined as inducing one of the "possible" distributions. As we no longer require that values of interpretation be expressions, what we have reached is the following "abstract" notion of interpretation: *an interpretation is a compositional assignment of objects to parameters leading to a "possible" distribution of truth values among statements*.

A kind of a by-product of the new, denotational notion of interpretation is the explication of the obejctual notion of instance encountered in the end of Section 3. We have concluded that the substitutional notion of an instance need not be in accordance with our intuition; that there may be objectual instances for which no corresponding substitutional instances obtain. Tarski's thesis has now led us to the needed criterion for the demarcation of the space of instances: there are just enough instances to make logical truth coincide with necessary truth.

Tarski's proposal, of course, was based on the inverse perspective: Tarski assumed that it is intuitively clear as to what the instances of a given schema are and that the concept of logical truth can thus be considered as a more or less empirical one. Hence the perspective is the one questioned by Wittgenstein (1984, I.8): "Die Logik ist eine Art von Ultra-Physik, die Beschreibung des 'logischen Baus' der Welt, den wir durch eine Art von Ultra-Erfahrung wahrnehmen (mit dem Verstand etwa)." But we deny (together with Wittgenstein, Quine and others) the possibility of inquiring into the world by-passing language, and hence we consider Tarski's perspective doomed. To know what the instances of a schema are is the same thing as to know what is possible and, therefore, what is necessary; and to know what is necessary is the same thing as to know what is necessarily true. Hence knowing instances does not precede knowing necessary truth; and it is thus more appropriate to consider necessary truth constitutive to the space of instances (and interpretations), not vice versa.

This means that the notion of interpretation that we have just reached does justice both to the notion of interpretation as dis-abstraction, and to the notion of interpretation as characterization of truth. The general principles governing this notion of interpretation are as follows: such an interpretation is an assignment of objects (the nature of which is irrelevant) to expressions such that it fulfils three principles. First, the value of the interpretation of a whole is to be "computable" from those of its parts; hence the interpretation is to be *compositional*. (Note that compositionality is a purely technical matter here - it guarantees that interpretation will be something we shall be able reasonably to work with.) Moreover, the simpler the "computation" of the value of interpretation of a whole from those of its parts, the better[11]. Second, among the values assigned to statements there are some distinguished values[12]. Third, the assignment should be as economic as possible, i.e. the fewer interpreting entities, the better[13].

6 INTERPRETATION AS MEANING ASSIGNMENT

Now we can return to the considerations we have started from, to the notion of interpretation as meaning assignment. We have stated that meanings are not to be found by a direct empirical investigation of the world, but we have not rejected the plausibility of considering meaning as an object[14]. The way in which meaning must be approached has been formulated in the frequently quoted sentence due to Lewis (1972, p.173): "In order to say what a meaning *is*, we may first ask what a meaning *does*, and then find something that does that." This is not to say that we give up the effort of revealing the right nature of meaning; this is the recognition of the fact that there is really nothing to meaning beyond what meaning does.

However, if we try to analyze what we are able definitely to say about meaning, we come to the conclusion that the assignment of meanings to expressions must be something very close to interpretation in the sense of the previous section. Thus, meaning, as we handle it intuitively, also seems to be a *compositional* matter[15]. Besides this, meaning is what can be called *verifounded*: difference in truth value clearly implies difference in meaning[16]. Third, there seems to be something like *Occam's razor*, something that pushes the number of meanings as far down as possible[17]. Meaning assignment can thus be identified with that interpretation (in the above sense) of our language which leads to that distribution of truth values among its statements which really obtains. Thus, it seems that the notion of interpretation reached above can do justice even to the notion of interpretation as meaning assignment.

It can be proved that a meaning assignment fulfils the above characterization (*compositionality*, *verifoundation* plus *Occam's razor*) if and only if sameness of meaning coincides with intersubstitutivity *salva veritate*[18]. This result should not be too surprising: the idea that the meaning of an expression is the contribution of the expression to the truth value of the statements in which it occurs is nothing new, it can be traced back to Frege and Wittgenstein (not to speak of Leibniz). What we have arrived at is especially close to the idea of semantics put forward by Davidson (1984): "I suggest that a theory of truth for a language does, in a minimal but important respect, do what we want, that is, give the meanings of all independently meaningful expressions on the basis of an analysis of their structure."

This is precisely what semantic interpretation in the sense spoken of here does: it distinguishes between truth and falsity and it propagates this distinction "on the basis of an analysis of structure" of complex expressions (i.e. by doing justice to the principle of compositionality plus the principle of Occam's razor) up to their ultimate parts. The meaning of an expression is thus the simplest thing which allows for a compositional characterization of truth.

However, should this mean that meaning is a matter of the actual distribution of truth values among statements and that it thus depends on contingent facts? Not at all; at least not for natural language. Every change in semantic interpretation is conditioned by a change of the truth value of a statement; this, however, in no way means that every change of the truth value of a statement would really bring about a change of semantic interpretation. It may well be the case that the only change of truth value that leads to a change of semantic interpretation is a change of the truth value of a *necessary* statement; and this is indeed the case of natural language. Natural language is inherently *intensional*; whereas some artificial languages (for example the traditional predicate calculus), for which semantic interpretation does depend on contingent truth, are *extensional*.

This throws quite a peculiar light on extensional languages, but the reason is simple: with respect to these languages it is simply not possible to speak about anything like meaning worth its name. Intensionality is an *essential* property of natural language; and extensional languages are from this point of view rather "pseudolanguages": they share some essential features with real languages, but not enough to be on a par with them in respect to meaning.

Extensional languages can, however, help us in clarifying another problematic semantic concept, namely that of *reference*. These languages, such as the classical predicate calculus, are results of the formalization of a certain restricted part of our language, of a part which is in a sense distinguishable from the rest of language. It is a kind of language within language[19], and it is a plausible hypothesis that what we call reference is just semantic interpretation of this extensional core of our language.

7 CONCLUSION

We have tried to show that the diverse senses in which the concept of *semantic interpretation* is used can be brought to one and the same notion. It is the notion of interpretation as an assignment of entities to expressions compositionally characterizing truth. Thus, a semantic interpretation can be seen as a certain kind of account for truth; as an account for which Tarski's idea of identification of *necessary truth* with *verification by every interpretation* is crucial. It is, however, necessary truth, i.e. the fact that we understand some statements as not capable of being false, that is basic: hence not "a statement is necessarily true *because* it is verified by every interpretation", but rather "a statement is necessarily true and *therefore* there cannot be an interpretation which falsifies it". Formal interpretation can be considered a matter of semantics because it accounts for necessary truth, not because it imitates a real denotandum/denotatum relation.

Every theory aims at an explication of some facts which are prior to it. To understand such a theory properly, it is essential to distinguish carefully between that which was here from the beginning and that which we built on top of it in our effort to "make sense" of it. The former can only be *described* (or in a way "systematized"), the latter can be *explained* (away) by pointing out its role in the pursuit of the description of the former. It is a basic error to try to explicate the former in the same way as the latter. We may explain the fact that expressions have different meanings by pointing out that they make different contributions to truth values of statements in which they occur, but we can hardly explicate the fact that there are statements which are true, or, using the material mode of speaking, that there is anything that is.

NOTES

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1. E.g. Robinson (1965).
2. Slight semantic differences - e.g. different felicity conditions - are likely to be found even between *John loves Mary* and *It is John who loves Mary*. Hence whether they really do boil down, as they usually do, to a single formula, depends on the threshold of difference we decide to take as significant.
3. It would be more accurate to call them *variable* formal expressions, but the term *variable* is traditionally used in a different sense within logic. Note the essential difference between variables and parameters: variables are tools of quantificational theory which are in fact unessential (it is possible - although a bit cumbersome - to do logic without them; see Quine, 1960 and also Peregrin, 1992a).

4. The boundary between these two kinds of truth is surely fuzzy (witness Quine's *Everything green is extended*). Moreover, as Wittgenstein and Quine stressed, it is in fact not an absolute matter, it is rather a matter relative to an adopted conceptual framework. However, despite both of these facts the boundary surely *is* something quite meaningful.
5. S is a necessary truth iff it is entailed by the empty set; and S is entailed by $S_1,...,S_n$ iff $(S_1\text{->}(...(S_n\text{->}S)...))$ is a necessary truth.
6. The idea, however, can be traced back to Bolzano. See Etchemendy (1990).
7. Note however, that such a notion of an individual is not a product of metaphysical speculation, but rather the outcome of our insistence on Tarski's thesis. Let us avoid the usual misguided idea that by giving an object a name we transform it into that which the name usually denotes. That we call what we have invented *individuals* is to keep within the boundaries of the usual logical terminology, it is not to attribute hands and feet to these entities!
8. A statement consisting of only logical constants is clearly inevitably accounted for as necessary: there are no alternative ways of interpreting it. However, this is appropriate at most in the case of the pure predicate calculus; it is enough to add **=** to it in the customary manner, and the picture gets distorted, as there emerge statements which consist purely of logical constants, which are, nevertheless, intuitively contingent. This is the case of $\exists x\exists y\neg(x=y)$ as discussed by Etchemendy.
9. Etchemendy's example implies that this is in fact the case with quantifiers. A simpler example is that of adding the axiom $\forall x(P(x)\text{->}Q(x))$ to the predicate calculus (for some definite predicate constants *P* and *Q*). *P* and *Q* are then no longer extralogicals: they are not replaceable by *any* other predicates. On the other hand, they are also not definite in the sense of **&**: they can be replaced, but only by some pairs of predicates and not by others.
10. To say that $S_1,...,S_n$ entail S is to say that it is impossible that S is false and $S_1,...,S_n$ are at the same time all true. See Peregrin (1992b).
11. Thus within the classical predicate calculus the interpretation of an atomic statement is not only uniquely determined by the interpretation of the predicate and the interpretation(s) of the term(s), but it is computable simply as the functional application of the former to the latter. The value assigned to a complex statement can again be computed as the application of the interpretation of the connective (which is the usual truth function) to the interpretations of the substatements.
12. In classical logic there is simply one distinguished and one undistinguished value, the case of modal logic, however, shows that a more general approach is appropriate.
13. If it were not for this third requirement, an identical mapping could be considered as an interpretation; and this is clearly absurd.
14. There are outstanding philosophers that *would* question this assumption - e.g. Austin or Quine.
15. Compositionality is also what underlies the most influential theories of meaning since Frege. In fact if we accept Frege's conviction that what is primarily meaningful are sentences, then we need the principle of compositionality in order to be at all able to individuate meanings of parts of sentences. It is, however, important to realize that understood this way compositionality is not a thesis to be verified or falsified, that it is rather a postulate which is constitutive to semantics.
16. Cresswell (1982) considers this the most certain principle of semantics.
17. "Senses are not to be multiplied beyond necessity", as Grice (1989, 47) puts it.
18. The problem can be formulated in algebraic terms: if we view language as a many-

sorted algebra (in the spirit of Janssen, 1983), then a meaning assignment fulfils the three conditions iff it is a homomorphism with its kernel equal to the maximal congruence for which it holds that no true statement is congruent with a false one. It is easy to see that it is just this congruence which coincides with the relation of intersubstitutivity *salva veritate*.

19. Its exceptional status manifests itself in the fact that many of the greatest philosophers of language, such as Frege, Wittgenstein, or Church, tried hard to restrict themselves to this very part. We can hardly ascribe to them an inability to see that there are intensional contexts in our language; rather they clearly considered these contexts as is in some sense secondary to the extensional core.

REFERENCES

Barwise, J. and Perry, J.: 1983, *Situations and Attitudes*, MIT Press, Cambridge (Mass.).

Cresswell, M.J.: 1973, *Logic and Languages*, Methuen, London.

Cresswell, M.J.: 1982, 'The Autonomy of Semantics', *Processes, Beliefs and Questions* (ed. by S.Peters and E.Saarinen), Reidel, Dordrecht.

Davidson, D.: 1984, *Inquiries into Truth and Interpretation*, Clarendon Press, Oxford.

Etchemendy, J.: 1988, 'Tarski on Truth and Logical Consequence', *Journal of Symbolic Logic* **53**, pp. 51-79.

Etchemendy, J.: 1990, *The Concept of Logical Consequence*, Harvard University Press, Cambridge/London.

Frege, G.: 1879, *Begriffsschrift*, Nebert, Halle.

Goodman, N.: 1960, 'The Way the World is', *The Review of Metaphysics* **14**, pp. 48-56.

Grice, P.: 1989, *Studies in the Ways of Words*, Harvard University Press, Cambridge (Mass.).

Janssen, T.M.V.: 1983, *Foundations and Applications of Montague Grammar*, dissertation, Mathematisch Centrum, Amsterdam.

Lewis, D.: 1972, 'General Semantics', *Semantics of Natural Language* (ed. by D.Davidson and G.Harman), Reidel, Dordrecht.

Lukasiewicz, J. and Tarski, A.: 1930, 'Untersuchungen Über den Aussagenkalkül', *Comptes Rendus des seances de la Societe des Sciences et des Lettres de Varsovie* **23**, pp. 30-50.

Peregrin, J.: 1992a, 'The Role of Variables in Logical Formalization of Natural Language', *Proceedings of the Eight Amsterdam Colloquium*, University of Amsterdam, pp. 463-481.

Peregrin, J.: 1992b, 'Meaning, Truth and Models', *From the Logical Point of View* 2/92, pp. 67-75.

Quine, W.V.O.: 1960, Variables explained away, *Proceedings of the American Philosophical Society* 104, pp. 343-347; reprinted in Quine: *Selected Logic Papers*, Random House, New York, 1966.

Quine, W.V.O.: 1963, *From the Logical Point of View*, Harper and Row, New York.

Quine, W.V.O.: 1969, *Ontological Relativity and Other Essays*, Columbia University Press, New York.

Robinson, A.: 1965, *Introduction to Model Theory and to the Metamathematics of Algebra*, North-Holland, Amsterdam.

Tarski, A.: 1936, 'Über den Begriff der logischen Folgerung', *Actes du Congrés International de Philosophie Scientifique* **7**, pp. 1-11.

Wittgenstein, L.: 1984, *Bemerkungen über die Grundlagen der Mathematik* (Werkausgabge Band 6), ed. by G.E.M.Anscombe, R.Rhees and G.H.von Wright, Suhrkamp, Frankfurt am Main.