Constructions and Concepts

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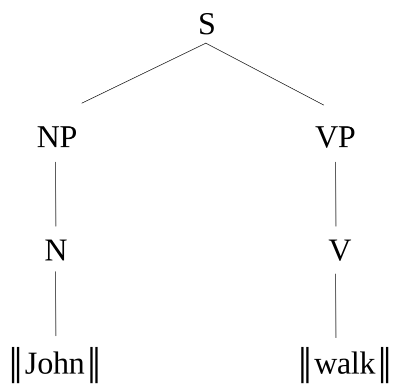
Tichý's hyperintensional semantics

Some twenty years ago, semanticists of natural language came to be overwhelmed by the problem of semantic analysis of *belief sentences* (and sentences reporting other kinds of propositional attitudes): the trouble was that sentences of the shapes *X believes that A* and *X believes that B* appeared to be able to have different truth values even in cases when *A* and *B* shared the same intension, i.e. were, from the viewpoint of intensional semantics, synonymous[1]. Thus, taking intensional semantics for granted, belief sentences appeared to violate the principle of intersubstitutivity of synonyms. The verdict of the gurus of intensional semantics was that hence intensional semantics is inadequate, or at least insufficient for the purposes of analysis of propositional attitudes; and that we need a kind of a 'hyperintensional semantics'.

The *locus classicus* of considerations of this kind is Lewis (1972): the author concludes that to be able to give a fair account of meaning of belief sentences we need an augmentation of the intensional semantics in the spirit of Carnap's (1957) notion of *intensional isomorphism*. Carnap's proposal amounts, in effect, to the notion of meaning according to which the meaning of a complex expression would be an ordered *n*-tuple of the intensions of its parts; so that if we denote the intension of *E* as ‖*E*‖, the meaning of (1) would be (1′):

John walks (1)
<‖John‖,‖walk‖> (1′)

Lewis improved on this suggestion by requiring that the meaning should somehow directly reflect the syntactic structure of the corresponding expression, so that the meaning of (1) would be not (1′), but rather something like (1″):



(1′)

[1] See, e.g., Bigelow (1978); and also Materna (1983).

This is a proposal that clearly solves the problem at hand; nevertheless the solution appears to be a bit too *ad hoc*.

Pavel Tichý, the author of *transparent intensional logic* (*TIL*) came with a similar proposal, which, however, was backed by a more intuitive story. The story goes as follows. According to extensional semantics, the meaning of (1) results from the application of the meaning of *walk* (a function mapping individuals on truth values) to that of *John* (an individual). According to intensional semantics, the situation is somewhat more complicated for all of this gets relativized to possible worlds; but again, the meaning of (1) results from a certain kind of combination of the meaning of *walk* (which now is a function mapping possible worlds on functions from individuals to truth values) with that of *John* (which may be a function mapping possible worlds on individuals; or, as Tichý had it, directly the individual named). Because this combination will play an important role for us later on, let us coin a name for it: *intensional application*. Thus, intensional application of an object to another object or other objects is like ordinary application save for an additional 'pre- and post-processing': all of the involved objects that are intensions (i.e. functions from possible worlds) get, prior to the application, applied to the variable *w*; and the result of the application then gets abstracted upon the very variable. (Later on, when Tichý started to relativize extensions not only to worlds, but also to time moments, the application and abstraction came to involve the variables *w* and *t* instead of merely *w*.) Now Tichý's suggestion is that although what a complex expression like (1) amounts to is this kind of intensional application, what should be seen as the meaning of the expression is not the *result* of such an application, i.e. an intension, but rather the very application itself - the *construction*, as he puts it.

Constructions

What is clearly crucially presupposed by Tichý's proposal is getting a firm grip on the concept of construction. What, then, is a *construction* in Tichý's sense? Does Tichý manage to explicate the concept in an acceptable and sufficient way?

The trouble is that for many decades the paradigm of a reasonable explication of a pre-theoretical entity (and at the same time the proof that the entity 'indeed exists') has been its reconstruction within set theory. Take, say, the concept of lattice: its canonical explication has come to be identified with a certain kind of algebra, i.e. a certain kind of ordered pair consisting of a set (the 'carrier' of the algebra) and a set of sets of ordered (n+1)-tuples of the elements of the carrier (the 'operations' of the algebra). Or take the concept of intension: it has come to be identified with a function taking possible worlds (and, as the case may be, time moments) to extensions (which are again construed as set-theoretic objects). Also when Lewis proposed his structured meanings mentioned above, he hastened to add that rigorously explained, they of course are certain set-theoretical entities. Tichý's ambition, on the other hand, was to somehow *forego* set theory: he assumed that his constructions were not to be accomodable within set theory, for they were to be more fundamental then anything like sets.

However, it is clear that not even every kind of abstract entity accepted by set-theoretically oriented mathematicians can be reduced to a set-theoretical construct. There is obviously at least one for which this is not possible, namely set itself. How, then, is the concept of set usually explicated? The standard way is the formulation of an axiomatic set theory, usually within the framework of first-order logic (although, e.g., second-order set theory is, of course, also conceivable). So would this not be also a way of explicating the concept of construction? Could we not try to articulate an axiomatic theory of constructions, analogous to set theory?

Tichý, to be sure, would not think so: for him, forming axiomatic theories of the kind of set theory was a useless formalistic game. (His standpoint was, in this point, similar to that of Frege in his well-known quarrel with Hilbert over the nature of geometry[2].) Hence, he purposefully tried to explicate the concept of construction on a less formal level. My opinion is, nevertheless, that indicating how an axiomatic theory of constructions might look like may be instructive, that it can help us throw some more light on the nature of Tichý's approach. Hence before I proceed to the central theme of the paper, Materna's explication of the concept of concept, I attempt to outline such a theory. I will try to 'translate' what Tichý says about the nature of constructions into the language of first-order logic and try to articulate it as an axiomatic theory. (However, it should be kept in mind that what I aspire to is nothing more than a sketch.)

Probably the first thing to notice when we try to form a 'construction theory' in the spirit of set theory is that whereas set theory does not presuppose anything but general logic (the only non-logical term of the formal theory is the predicate of set membership), construction theory presupposes some nontrivial concepts, namely the concepts of *function* and *functional application*. Explaining the nature of constructions, Tichý takes these concepts simply for granted. Thus, we have to assume that the language in which we formulate the theory contains not only logical primitives, but rather also, for every natural number n, the unary predicate Fnc^n and the (n+1)-ary predicate Appl^n together with some axioms fixing the intended meanings of $\mathrm{Fnc}^n(x)$ to „x is an n-ary function" and $\mathrm{Appl}^n(y,x_1,\dots x_n,z)$ to „z is the result of the application of y to $x_1,\dots, x_n$"[3]. (We leave it open precisely what kind of theory this is supposed to be. Clearly one option would be to let the construction theory be underlain by set theory with *its* explication of the concepts of function and application; but this would obviously frustrate Tichý's effort to get *beyond* set theory.)

So given we have a theory of functions, a way of building a theory of constructions atop of it might be the following. Tichý speaks about seven types of constructions, so we should have five unary predicates true of all and only constructions of the respective types. Let the predicates be Var („is a variable"), Triv („is a trivialization"), Exe („is an execution"), DExe („is a double execution"), Comp („is a composition") and Clos („is a closure"). We can define the predicate Cons („to be a construction") as

$$\mathrm{Cons}(x) \equiv_{\mathrm{Def}} \mathrm{Var}(x) \lor \mathrm{Triv}(x) \lor \mathrm{Exe}(x) \lor \mathrm{DExe}(x) \lor \mathrm{Comp}(x) \lor \mathrm{Clos}(x)$$

Moreover, as we assume that nothing can instantiate two different types of constructions, we should have axioms of the kind of

$$\forall x\, (\mathrm{Var}(x) \to \neg(\mathrm{Triv}(x) \lor \mathrm{Exe}(x) \lor \mathrm{DExe}(x) \lor \mathrm{Comp}(x) \lor \mathrm{Clos}(x))$$
$$\forall x\, (\mathrm{Triv}(x) \to \neg(\mathrm{Var}(x) \lor \mathrm{Exe}(x) \lor \mathrm{DExe}(x) \lor \mathrm{Comp}(x) \lor \mathrm{Clos}(x))$$
...

[2] See Frege (1976). See also Peregrin (to appear a; §4).

[3] We could also make do with taking only Appl^n as primitive and define $\mathrm{Fnc}^n(y)$ as $\exists x_1 \dots x_n z\ \mathrm{Appl}^n(y,x_1,\dots x_n,z)$; but this is not important now.

How are individual constructions to be characterized? Constructions *construct*, so we should have a relation connecting them to what they construct. However, some of the constructions (variables and open constructions) construct something only relative to a valuation of variables. It follows that in order to be able to address constructing, we first have to deal with *v-constructing*, i.e. constructing relative to a given valuation. And hence it follows that before we are able to address constructing in general, we have to address one specific kind of constructions, namely variables.

1. Variables. A variable is an entity characterized simply by the fact that it constructs (i.e., expressed more traditionally, has) a value, relative to a given 'valuation'. Hence to characterize variables within our axiomatic system, we have to assume that our universe contains, in addition to variables themselves, also entities called *valuations*, which somehow 'make' variables acquire objects of the universe as their values. (Don't be puzzled by the fact that we do not treat variables as linguistic items and valuations as 'metalinguistic' ones; the 'objectual' treatment of variables is one of the key points of Tichý's approach. And don't confuse the variables of TIL, which thus come to be objects of the universe of our theory, with the variables which are part of the first-order language by means of which we articulate the theory.) One of the ways of axiomatically reflecting this is introducing the unary predicate Val („is a valuation") and a ternary relation Value („the value which the valuation ... assigns to the variable ... is ...") governed by at least the following axioms:

$\forall vxy\ (\mathrm{Value}(v,x,y) \rightarrow (\mathrm{Val}(v)\wedge\mathrm{Var}(x))$

$\forall vx((\mathrm{Val}(v)\wedge\mathrm{Var}(x)) \rightarrow \exists y\mathrm{Value}(v,x,y))$

$\forall vxyy'((\mathrm{Value}(v,x, y)\wedge\mathrm{Value}(v,x,y')) \rightarrow (y=y'))$

$\forall x_1...x_ny_1...y_n((\mathrm{Var}(x_1)\wedge...\wedge\mathrm{Var}(x_n)) \rightarrow$
$\qquad \exists v(\mathrm{Val}(v)\wedge(\mathrm{Value}(v,x_1,y_1)\wedge ...\wedge(\mathrm{Value}(v,x_n,y_n)))$

$\forall vv'((\mathrm{Val}(v)\wedge\mathrm{Val}(v')) \rightarrow(\forall xyy'((\mathrm{Value}(v,x, y)\wedge\mathrm{Value}(v',x,y')) \rightarrow (y=y')) \rightarrow (v = v')))$

The first of the axioms states that what assigns values are valuations whereas what is assigned values are variables[4]. The next two axioms state that for every variable and every valuation there is one and only one object assigned to the variable by the valuation. The fourth axiom states that for every n-tuple of variables and every n-tuple of objects there exists a valuation assigning the objects to the variables. The last axiom then states that two valuations which assign the same objects to the same variables are identical.

Given this, we can introduce the ternary relation VConstr („with respect to the valuation ..., ... constructs ...") such that it always holds between a valuation, a construction and an object, and it can be seen as inducing a (partial) function assigning its last argument to its first two:

$\forall vxy\ (\mathrm{VConstr}(v,x,y) \rightarrow (\mathrm{Val}(v) \wedge \mathrm{Cons}(x)))$.

$\forall vxyy'\ ((\mathrm{VConstr}(v,x,y) \wedge\mathrm{VConstr}(v,x,y')) \rightarrow (y = y'))$.

The relation is to be further characterized specifically for individual types of constructions, to which we can now turn our attention.

[4] The axiom could incorporate also some restriction with respect to the kind of objects assigned to variables by valuations: it might be, e.g. excluded that an object assigned to a variable is a valuation.

2. Trivialization. As trivialization is always 'of something', we need a functor to take us from this something to the respective trivialization. So let Triv^* be an unary functor such that[5]

$$\forall x\ \mathrm{Triv}(\mathrm{Triv}^*(x)).$$

A further axiom should probably guarantee that Triv^* is injective:

$$\forall xy\ ((\mathrm{Triv}^*(x) = \mathrm{Triv}^*(y)) \rightarrow (x = y))$$

The most essential axiom concerning trivialization now would be one spelling out that a trivialization of an entity constructs the very entity:

$$\forall vxy\ (\mathrm{VConstr}(v,\mathrm{Triv}(x),y) \leftrightarrow (y=x))$$

3. Execution. An execution is, like a trivialization, of something; so we introduce the predicate Exe^* and stipulate

$$\forall x\ \mathrm{Exe}(\mathrm{Exe}^*(x)).$$
$$\forall xy\ ((\mathrm{Exe}^*(x) = \mathrm{Exe}^*(y)) \rightarrow (x = y))$$

The crucial axiom here is one stating that an execution of a construction constructs what is constructed by the very construction (if anything):

$$\forall vxy\ \mathrm{VConstr}(v,\mathrm{Exe}^*(x),y) \leftrightarrow \mathrm{VConstr}(v,x,y)$$

4. Double Execution. The situation is again analogous:

$$\forall x\ \mathrm{DExe}(\mathrm{DExe}^*(x)).$$
$$\forall xy\ ((\mathrm{DExe}^*(x) = \mathrm{DExe}^*(y)) \rightarrow (x = y))$$

Here the crucial axiom spells out that a double execution of a construction constructs what is constructed by what is constructed by the construction (if anything):

$$\forall vxy\ (\mathrm{VConstr}(v,\mathrm{DExe}^*(x),y) \leftrightarrow \exists z\ (\mathrm{VConstr}(v,x,z) \wedge \mathrm{VConstr}(v,z,y)))$$

5. Composition. Here the situation is more complicated, for composition is of more than one object; we can have a composition of *n* objects for any natural n>1. So what we need is an (n+1)-ary functor Comp^n for every natural *n*, governed by the axioms of the following kind:

$$\forall yx_1...x_n\ \mathrm{Comp}(\mathrm{Comp}^n(y,x_1,...,x_n))$$

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[5] Of course that we could *define* Triv in terms of Triv^*, *viz* $\mathrm{Triv}(x) \equiv_{\mathrm{Def}} \exists y(x = \mathrm{Triv}^*(y))$; but again we do not dwell on such details.

$$\forall v y x_1 \ldots x_n z v' y' x_1' \ldots x_n' \, (\mathrm{VConstr}(v,\mathrm{Comp}^{n}(y,x_1,\ldots,x_n),z) \leftrightarrow (\mathrm{VConstr}(v,y,y') \wedge \mathrm{VConstr}(v,x_1,x_1') \wedge \ldots \wedge \mathrm{VConstr}(v,x_n,x_n') \wedge \mathrm{Appl}^{n}(y',x_1',\ldots,x_n',z)))$$

The last axiom states that a composition constructs the result of application of what is constructed by its first argument to what is constructed by the rest of the arguments.

6. Closure. We again need, for every natural n, an $(n+1)$-ary functor Clos^{n}:

$$\forall y x_1 \ldots x_n \, ((\mathrm{Var}(x_1) \wedge \ldots \wedge \mathrm{Var}(x_n)) \rightarrow \mathrm{Clos}(\mathrm{Clos}^{n}(x_1,\ldots,x_n,y)))$$

$$\forall v y x_1 \ldots x_n z \, (\mathrm{VConstr}(v,\mathrm{Clos}^{n}(x_1,\ldots x_n,y),z) \leftrightarrow (\mathrm{Var}(x_1) \wedge \ldots \wedge \mathrm{Var}(x_n) \wedge \forall v' a_1 \ldots a_n \, (\mathrm{Val}(v') \wedge (\mathrm{Value}(v',x_1,a_1) \wedge \ldots \wedge (\mathrm{Value}(v',x_n,a_n) \wedge \forall x(((x \neq x_1) \wedge \ldots \wedge (x \neq x_n)) \rightarrow \forall w(\mathrm{Value}(v',x,w) \leftrightarrow \mathrm{Value}(v,x,w)))) \rightarrow \exists u \, (\mathrm{Appl}^{n}(z,a_1,\ldots,a_n,u) \wedge \mathrm{VConstr}(v',y,u)))$$

Here the last axiom spells out the fact that a closure is a way of turning a construction of an object into that of a function: $\mathrm{Clos}^{n}(x_1,\ldots,x_n,y)$ constructs the function which applied to the n-tuple $a_1,\ldots,a_n$, yields what would be constructed by the construction y if the variables $x_1,\ldots,x_n$ constructed $a_1,\ldots,a_n$, respectively.

This completes our sketch of an axiomatic theory of constructions. I think that despite the fact we have refrained from dwelling on details (which would, of course, become important were somebody to take the task of constructing this kind of theory seriously), our exercise has indicated that it does not seem to be impossible to explicate the concept of construction in this way, i.e. in the usual axiomatic manner. However, the resulting theory appears to be quite different from set theory: it is not so ground level as set theory is (it presupposes the concepts of function and functional application), it is much less 'elegant', and it also does not seem to give rise to nontrivial mathematical problems in the way set theory does (at least not directly).

Systems of constructions

Tichý's system of constructions (TSC, for short), as we have just seen, is based on six types of constructions. An interesting question now is: Why should we accept that there are just the constructions he claims there are?

Suppose I would insist that there is a peculiar kind of construction called *addition* which turns a pair of constructions of natural numbers into a construction of the sum of the numbers. More precisely, if C_1 and C_2 are constructions such that C_1 constructs the natural number n_1 and C_2 constructs the natural number n_2, then $[C_1\ C_2]_+$ constructs the number n+m (and it constructs nothing if either C_1 or C_2 does not construct a natural number). Tichý would probably respond that what I see as a peculiar kind of construction is in fact a specific case of his composition, that what I see as $[C_1\ C_2]_+$ is 'in fact' his $[^0+\ C_1\ C_2]$, i.e. the application of the trivialization of + to C_1 and C_2. However, how can one justify a claim that one construction is *in fact* another construction? We can claim, to be sure, that one system of constructions can be reduced to another system of constructions - but why should we say that the constructions of the former are therefore 'in fact' those of the latter?

An argument against my proposal might, of course, be based on the fact that if we had a separate construction for addition, we would need one for subtraction, multiplication etc., and that this would lead to a proliferation of constructions. Thus, whereas TSC makes do with altogether six types of constructions, what I have proposed would yield a many times more numerous denizens of constructions. Thus, if what we are after is a *minimalist* stock of basic primitives, TSC is undoubtedly better[6]. However, what if we proposed a system of constructions which is different from TSC, but equally simple? Let us consider two such proposals:

1. It is, it seems to me, slightly counterintuitive to say that the expression „1+2" expresses a construction of the application of *the trivialization of* + to *the trivializations of* 1 and 2: it would seem to be more natural to say that it is simply the construction of application of *the object* + to *the objects* 1 and 2. My opinion is that the source of this counterintuitivness is the fact that Tichý's construction of composition meshes together two intuitively different constructions: the construction of application proper (just exemplified) and the construction of composition of constructions. An example of the latter would be the application of + to the results of some sub-operations, like in the construction expressed by „(1+2)+(3+4)".

In view of this fact we might attempt to improve on TSC by replacing Tichý's composition by the following two constructions:

Application: If f is an n-ary function and $x_1,...,x_n$ are objects, then (f x_1 ... x_n) is a construction constructing the value of f for the arguments $x_1,...,x_n$.

Composition: If C is a construction of the object x from the objects $x_1,...,x_n$ and C′ is the constructions of x_i from $x_1^i,..., x_m^i$, then the result of replacing x_i in C by C′ is a construction of x from $x_1, ..., x_{i-1}, x_1^i,..., x_m^i, x_{i+1},...,x_n$.

Now the construction expressed by „1+2" would be simply (+ 1 2), while that expressed by „(1+2)+(3+4)" would be (+ (+ 1 2)(+ 3 4)).

So here we have an example of a rival system of constructions: it has one more basic construction than TSC, but the gain is that it might be, at least in some respects, more conceptually perspicuous. (Moreover, the new system could in fact not need more types of constructions that TSC, for I suspect that once we have the above construction of application, we no longer need trivialization. But this is not a theme for us now.)

2. The second example I am going to present is more important for our subsequent considerations. There is obviously a close parallel between the structure of TSC and that of the two-sorted variant of the language of typed lambda-calculus of Church (1940) - in fact, it seems as if the two-sorted lambda-calculus were just the canonical language expressing the constructions of TSC. However, Church's typed lambda calculus is known to be translatable into combinatory logic[7] - so why not consider the system of constructions for which combinatory logic would be the canonical language in the very sense in which two-sorted type theory is for TSC? If we call this system of constructions the *combinatory system of constructions*, *CSC*, we may ask: what makes TSC the 'right' system and CSC a 'false' one? And as far as I can see, the only feasible answer is that TSC appears to be, in some sense,

[6] Let me note in passing that it seems that Platonistically-minded logicians like Tichý and Materna do *not* appear to be after something like „the smallest store of materials with which a given logical or semantic edifice can be constructed" (Russell, 1914, p.51) - what they do seem to be after is rather the discovery of the matter-of-factual system of constructions underlying what we say.

[7] See Curry and Feys (1957).

more handy. However, in what follows we will see that ridding ourselves of variables, which is what the replacement of TSC by CSC would effect, might be a useful thing. However, to see this, we must consider further aspects of TIL.

Constructions and natural language expressions

Another important question related to TSC concerns the criterion for deciding which construction is expressed by a given natural language statement. Let us restrict ourselves to sentences and let us ask: how should we tell which construction is expressed by a given sentence?

Notice that if we believed, as the (pre-hyper-)intensional logicians did, that what is expressed by a sentence is (capturable as) a class of possible worlds[8], the criterion would be clear: a given sentence would express the class of those and only those possible worlds in which it is true. It is obvious that the criterion pins down a single unique entity (disregarding, of course, the vagueness of natural language sentences): there is one and only one class of those possible worlds in which a sentence is true. This criterion is still usable as a constraint even if we now claim that what sentences express are constructions: a given sentence is bound to express a construction constructing the class of those and only those possible worlds in which the sentence is true. However, this constraint now no longer pins down a unique entity: for every construction, there clearly exists an infinite number of different constructions equivalent to it in the sense of constructing the same class of possible worlds. Which one, then, is *the* construction expressed by the sentence?

Let us take an example; let us consider the sentence

Venus is a planet. (2)

According to Materna (1998, p. 43), this sentence expresses the construction

$$\lambda wt.[{}^0\mathbf{planet}_{wt}\ {}^0\mathbf{Venus}], \qquad (2')$$

where **planet** is an object of the type (οι)τω and **Venus** is an object of the type ι. Now there is a number of constructions equivalent to (2′), among others

$$\lambda wt.[\ {}^0\mathbf{\&}\ [{}^0\mathbf{planet}_{wt}\ {}^0\mathbf{Venus}]\ [{}^0\mathbf{=}\ [{}^0\mathbf{+}\ {}^0\mathbf{1}\ {}^0\mathbf{1}]\ {}^0\mathbf{2}]]], \qquad (2^*)$$

where **&** is the usual conjunction (the object of the type (οοο)); or

$${}^0\mathbf{Venus\text{-}is\text{-}a\text{-}planet}, \qquad (2^{**})$$

where **Venus-is-a-planet** is a suitable object of the type οτω; or

$$[{}^0\mathbf{is\text{-}a}\ {}^0\mathbf{planet}\ {}^0\mathbf{Venus}] \qquad (2^{***})$$

[8] Or, better, a (generally partial) function from possible worlds to truth values.

where **is** is a suitable of object of the type (οτω)((οι)τω)ι. Why should we say that it is (2′), and not (2^*), (2^{**}) or (2^{***}), that is expressed by (2)?

The only way of answering this question I can see is that the appropriateness of (2′) somehow rests on the (syntactic) structure of (2). That is to say that (2′) is a candidate for the construction expressed by (2) superior to, e.g., (2^*), because (2^*) contains numbers which are not mentioned at all in (2). A similar argument can be used if we want to justify the superiority of (2′) over (2^{**}): (2^{**}) consists of a single component, whereas (2) is a compound, it contains more than one term. However, then it would seem that a wholly analogous argument could be used to show that (2^{***}) is superior to (2′): for (2^{***}) contains an object corresponding to the part 'is' of (2) which is lacking in (2′). (And of course it would be possible to further refine (2^{***}) into a construction containing separate parts for 'is' and 'a'.)

Imagine that we accept a system of constructions, different from TSC, which contains the following type of construction:

Basic intensional application. If *P* is an (οι)τω-object and *T* a ι-object, then {*T* is a *P*} constructs a οτω-object which takes *w* and *t* into *truth* just in case *T* belongs to $P_{wt.}$ (The string 'is a' is to be considered as a syncategorematic sign akin to the brackets '{' and '}'.)[9]

Given this definition, we could analyze (2) by means of

{**Venus** is a **planet**},

the structure of which is in the straightforward correspondence with the syntactic structure of (2).

So it seems that if we accept that a construction is the better expression of what a given sentence says the closer it is to the syntactical structure of the sentence, our last proposal would fare better than the standard TSC-based analysis. Moreover, it would be generally preferable to switch from TSC to CSC - natural language expressions do not contain anything like variables and so the constructions of TSC, unlike those of CSC, are bound to be structurally deviant from natural language expressions.

Materna's way from constructions to concepts

Materna, who has been interested in the concept of concept for a long time, realized that constructions can well serve for the purposes of explicating this concept. In fact, closed constructions are something very close to what concepts, as he uses the term, are[10]. The only problem which Materna saw was that constructions are a bit more fine-grained than concepts are supposed to be. This can be seen from the fact that, e.g., (3) and (3′) are different

[9] Note that this construction can be easily interpreted within TSC: from the viewpoint of Tichý's system, {*T* is a *P*} would 'in fact' be $\lambda wt.[P_{wt}\, T]$

[10] The term „concept" is sometimes interpreted as almost generally synonymous with „meaning", while sometimes it is rather interpreted more narrowly, as something like „meaning of a predicative expression". Whereas Frege, as is well known, endorsed the latter interpretation, Materna sticks to the former one, which, as he argues, was endorsed also, e.g., by Bolzano.

constructions, but intuitively they amount to one and the same concept, namely the concept of addition:

$$\lambda xy\ [^0\!+ x\ y] \quad (3)$$
$$\lambda yx\ [^0\!+ y\ x] \quad (3')$$

The remedy Materna proposes is to explicate concepts not directly as constructions, but rather as certain equivalence classes of constructions. For this purpose he defines two equivalence relations among constructions, namely the *α-equivalence* and the *β-equivalence*. Two constructions are *α-equivalent* iff one can be obtained from the other by systematic renaming of its lambda-bound variables; while they are *β-equivalent* iff one can be obtained from the other by erasing 'idle lambda-abstractions', rewriting λxP(x) as P. The relation of *quasiidentity* is then the transitive closure of the union of the relations of α- and β-equivalence. Eventually, a concept is the equivalence class of closed constructions according to the quasiidentity.[11]

I think the trouble with this move is that it largely spoils the intuitiveness of Tichý's picture and introduces an *ad hoc* element of the kind of that which impairs Lewis' theory. It seems to me that the basic attractiveness of Tichý's proposal consists in the fact that it is plausible to assume that the complex expressions of our language express ways in which meanings or concepts associated with their parts add up and construct something - and once we start to say that what the expressions express are in fact not constructions, but rather classes, the illuminating picture is gone. What I think would be a more promising route is to develop an alternative notion of construction which would not be 'over-fine-grained', i.e. which would enable us to identify concepts directly with constructions.

A moment reflection reveals that what causes the troubles Materna's definition of concept is devised to overcome are variables. No two different variable-free constructions are either α- or β-reducible. So if we restricted ourselves to variable-free constructions, we could identify concepts directly with constructions. And we have indicated above that there might be a way to accomplish this.

Let us return to our sentence (2):

Venus is a planet (2)

We have seen that there does not appear to be a decisive reason not to see it as expressing the construction

$$[^0\mathbf{is\text{-}a}\ ^0\mathbf{planet}\ ^0\mathbf{Venus}], \quad (2^{***})$$

or, allowing ourselves of the above construction of basic intensional application, as

{**Venus** is a **planet**}.

Now this can be generalized. Let us introduce the following type of construction:

[11] For rigorous definitions, see Materna (*op. cit.*, §5.3).

(General) intensional application. Let I be a subset of the set $\{0,...,n\}$. Let C_0, $C_1,...,C_n$ be such constructions that C_0 constructs either an object of the type $((\beta\alpha_1...\alpha_n)\tau\omega)$ or an object of the type $(\beta\alpha_1...\alpha_n)$, depending on whether $0\in I$ or $0\notin I$; and similarly every C_i constructs either an object of the type $(\alpha_i\tau\omega)$ or an object of the type α_i, depending on whether $i\in I$ or not. Then $[C_0\ C_1\ ...\ C_n]_I$ constructs an object of the type $(\beta\tau\omega)$ such that its value for a world W and a time moment T is the result of the application of the function O_0 to the objects $O_1,...,O_n$, where O_i is the object constructed by C_i if $i\in I$ and it is the value of the object constructed by C_i for W and T otherwise. (The equivalent of $[C_0\ C_1\ ...\ C_n]_I$ in TSC is $\lambda wt.[C_0^*\ C_1^*\ ...\ C_n^*]$ where X^* is X_{wt} for $i\in I$ and is X otherwise.)

Let us illustrate the definition by examples of instances of intensional applications with their equivalents within TSC:

[**planet Venus**]$_{\{0\}}$ $\lambda wt.[^0\mathbf{planet}_{wt}\ {}^0\mathbf{Venus}]$
[**finds John a-unicorn**]$_{\{0,2\}}$ $\lambda wt.[^0\mathbf{finds}_{wt}\ {}^0\mathbf{John}\ {}^0\mathbf{a\text{-}unicorn}_{wt}]$
[**seeks John a-unicorn**]$_{\{0\}}$ $\lambda wt.[^0\mathbf{seeks}_{wt}\ {}^0\mathbf{John}\ {}^0\mathbf{a\text{-}unicorn}]$
[**&** [**planet Venus**]$_{\{0\}}$ [**finds John a-unicorn**]$_{\{0,2\}}$]$_{\{1,2\}}$
$\lambda wt.[^0\mathbf{\&}\ [^0\mathbf{planet}_{wt}\ {}^0\mathbf{Venus}]\ [^0\mathbf{finds}_{wt}\ {}^0\mathbf{John}\ {}^0\mathbf{a\text{-}unicorn}_{wt}]]$

Let us return to the step which led from extensional semantics to the intensional one. Within extensional semantics, we have (besides others) subject terms meaning individuals, predicates meaning functions from individuals to truth values and sentences meaning truth values. So the meaning of a predicate and that of a term fit nicely together, in the sense that the former gets simply applied to the latter and what results is the meaning of their combination. Now the intensionalization of this model, which has turned out to be necessary to make the model at least minimally semantically plausible, threatened to spoil this nice fit. Within the intensional model, meanings get relativized to possible worlds (and, as the case may be, time moments), so that the meaning of a term now is a function from worlds to individuals, that of a predicate a function from worlds to the functions from individuals to truth values and that of a sentence a function from worlds to truth values. Hence the meaning of a predicate can no longer be simply applied to the meaning of a term to produce the meaning of the corresponding sentence.

What Tichý employed to save the situation were variables for possible worlds: the mechanism of lambda-abstraction, which was part of the apparatus anyway, then did all the rescue work, via the 'pre- and post-processing' mentioned above. (Montague originally attempted to do it differently, but his followers soon found out that to assimilate intensional logic to the two-sorted lambda-calculus, as Tichý, in effect, did from the beginning, is the most efficient way[12]). However, if we accept a new kind of construction as the intensional application defined above, we solve the problem bypassing the engagement of variables. And this may be, as we saw, convenient.

[12] Cf. Gallin (1975).

Logic without variables?

The indicated way of banishing the variables *w* and *t* from the usual constructions is only a specific case of a much more general mechanism (which, in effect, underlies the transformation of lambda-calculus into combinatory logic). Within categorial grammar, which underlies logics like TIL, you can only combine an expression of the type $C/C_1 \ldots C_n$ with expressions of the types $C_1, \ldots, C_n$. Now imagine you want to combine an expression of the type C_1/C_2 with an expression of the type C_2/C_3 into an expression of the type C_1/C_3. (Imagine, e.g., that you want to combine an (οο)-expression, like negation, with an (οι)-expression.) With the help of lambda-abstraction, you can accomplish this by, first, applying the latter expression to a variable of the category C_3, then applying the former expression to the result, and then abstracting the result upon the variable used in the first step. The question, though, is why not allow for a *direct* combination, especially when the combination makes a clear semantic sense: it amounts to composition of functions.

Hence although the traditional categorial grammar allows for the direct expression of only functional application, it would be straightforward to extend it in such a way that it would allow for expressing also all kinds of functional compositions. (Our intensional application would then be one of the possible grammatical rules of such an extended language.) In fact, ideas of this kind were expressed as early as in the fifties by Lambek (1958), and have come to flavor especially during the last decade[13].

Does, then, what I propose amount to banishing variables from logic altogether? Yes and no. The fact is that we *can* do logic without variables. It can be shown that any 'reasonable' logical calculus can be reformulated without the employment of variables. Quine (1960) has envisaged this for first-order predicate calculus; and this kind of treatment can be generalized (see Peregrin, to appear b, for some details). And, as I have tried to show, such a reformulation can be more than a formalistic game - it may be a way of gaining conceptual clarity.

However, I do not suggest that we should simply *forget* about variables - variables are hardly disapensable technical means of doing logic. What we, I think, should do, is to „speak with the vulgar, but think with the learned", i.e. to continue using variables, only not counting them to the essential, 'categorematic' inventory of our language, but seeing them rather as dispensable items on par with, say, brackets. We know, due to the Polish logicians, that brackets are dispensable, but we keep using them, for it is convinient - but we are not tempted to claim that they belong to our language in the same sense as predicates or logical connectives do. And we should, I suggest, see variables in an analogous way: employ them as an efficient technical tool, but disregard them when we ponder foundational questions such as the nature of concepts.

All of this is, of course, connected to the very way one sees the enterprise of logical analysis. What I am convinced is that we should see it not as a '(quasi)metaphysical' enterprise of discovering and reporting entities 'making up' the meanings of expressions, but rather as an *explicative* enterprise of envisaging the semantic, especially inferential, properties of expressions by means of building formal models (see Peregrin, 1998). I am convinced that the metaphysical stance entraps us into the net of really unanswerable (pseudo)questions like *Are variables really out there, within the meanings?*. I think that in contrast to this, the explicative stance allows us to replace such questions by much clearer and more contentful

[13] See, e.g., Morrill (1994).

ones, like *Is a model with variables in some sense more useful (e.g. more comprehensible, or more easy to handle) than one without them?*

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